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Joseph Helszajn (M'64) was born in Brussels, Belgium, in 1934. He received the Full Technological Certificate of the City and Guilds of London Institute from Northern Polytechnic, London, England in 1955, the M.S.E.E. degree from the University of Santa Clara, CA, in 1964, the Ph.D. degree from the University of Leeds, Leeds, England, in 1969, and the D.Sc. degree from Heriot-Watt University, Edinburgh, Scotland, in 1976.

He has held a number of positions in the microwave industry. From 1964 to 1966, he was Product Line Manager at Microwave Associates, Inc., Burlington, MA. He is now Professor of Microwave Engineering at Heriot-Watt University. He is the author of the books *Principles of Microwave Ferrite Engineering* (New York: Wiley, 1969), *Nonreciprocal Microwave Junctions and Circulators* (New York: Wiley, 1975), and *Passive and Active Microwave Circuits* (New York: Wiley, 1978).

Dr. Helszajn is a Fellow of the Institution of Electronic and Radio Engineers (England) and a Fellow of the Institute of Electrical Engineers. In 1968, he was awarded the Insignia Award of the City and Guilds of London Institute. He is an Honorary Editor of *Microwaves, Optics and Antennas* (IEE Proceedings).

Cyclotron Maser and Peniotron-Like Instabilities in a Whispering Gallery Mode Gyrotron

PETER VITELLO

***Abstract*—The efficiency of the mth harmonic electron cyclotron maser interaction for a TE_{mn1} gyrotron oscillator is compared with the $(m-1)$th harmonic peniotron-like interaction. Identical cavities and electron beams are used. Start oscillation conditions from weak-field linear theory are given, as well as optimized nonlinear efficiencies. The peniotron-like interaction leads to optimized efficiencies of ≤ 65 percent, while those for the electron cyclotron maser interaction are limited to ≤ 25 percent in the cases studied.**

THE ELECTRON CYCLOTRON maser interaction in gyrotron devices provides an extremely efficient mechanism for generating high-power microwave radiation [1]–[6]. In the cyclotron maser interaction, electromagnetic (RF) waves in a cavity or waveguide azimuthally bunch an electron beam in which the individual electrons move along helical orbits in the presence of an applied magnetic field. Numerous reviews and theoretical treatments of this instability have already appeared in the literature [1]–[5], [7]–[12]. While the cyclotron maser interaction has been shown capable of generating high microwave power, it is not the sole interaction which takes place in a gyrotron, nor is it necessarily the most efficient.

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The author is with Science Applications, Inc., McLean, VA 22102.

In this paper, we compare the cyclotron maser interaction with the less well-known [7], [12]–[15] peniotron-like interaction, for a whispering gallery mode TE_{mn1} gyrotron oscillator cavity. Emphasis will be placed on large values of m, and n equal to 1 or 2. The system considered consists of an axis-encircling electron beam (initially centered on the axis) in a right cylindrical cavity of length L and radius R, with a circularly polarized RF standing wave of amplitude E_O and a constant axial guide magnetic field B_O. The initial beam velocities, normalized to the speed of light c, parallel to and perpendicular to the axis are $\beta_{\parallel O}$ and $\beta_{\perp O}$. We will use dimensionless units, with the cavity radius as our scaling parameter. In these units, length is measured in

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units of R, time in units of R/c, and fields in units of $m_e c^2/|e|R$, where m_e is the electron mass and e the electron charge.

The TE_{mnl} RF fields in the cavity are given by

$$E_r = E_O\left(\frac{mJ_m(y)}{y}\right)\sin(k_{\|}z)\cos(m\theta - \omega t) \quad (1)$$

$$E_\theta = -E_O J_m'(y)\sin(k_{\|}z)\sin(m\theta - \omega t) \quad (2)$$

$$B_r = -E_O\left(\frac{k_{\|}}{\omega}\right)J_m'(y)\cos(k_{\|}z)\cos(m\theta - \omega t) \quad (3)$$

$$B_\theta = E_O\left(\frac{k_{\|}}{\omega}\right)\left(\frac{mJ_m(y)}{y}\right)\cos(k_{\|}z)\sin(m\theta - \omega t) \quad (4)$$

$$B_z = -E_O\left(\frac{X_{mn}}{\omega}\right)J_m(y)\sin(k_{\|}z)\cos(m\theta - \omega t) \quad (5)$$

where $k_{\|} = \pi/L$, $y = X_{mn}r$, X_{mn} is the nth nonvanishing root of $J_m' = O$, J_m is Bessel's function, and $\omega = (k_{\|}^2 + X_{mn}^2)^{1/2}$ is the cavity frequency.

The cyclotron maser interaction is relativistic in nature, being caused by the variation of electron mass with energy. For an axis-centered beam interacting with a TE_{mnl} mode, the cyclotron maser interaction takes place at the mth harmonic of the cyclotron frequency $\omega \simeq m\Omega/\gamma$, where Ω/γ is the cyclotron frequency, and γ is the Lorentz factor. At the mth harmonic, the electrons and rotating circularly polarized RF fields are synchronous. Near $\omega \simeq m\Omega/\gamma$, the efficiency η, defined as the ratio of the loss in beam kinetic energy to its initial energy, can be given in the weak-field linear limit for fast-wave interactions ($\omega \gg k_{\|}$) as

$$\eta \simeq \frac{E_O^2}{(k_{\|}\beta_{\|O})^2}\frac{J_m'(y)}{\gamma_O(\gamma_O - 1)} \cdot\left[\frac{\beta_{\perp O}^2\omega}{k_{\|}\beta_{\|O}}J_m'(y)g'(x) - (2\beta_{\perp O}mJ_m''(y) + 2J_m'(y))g(x)\right] \quad (6)$$

where $x = [(m\Omega/\gamma_O) - \omega]/k_{\|}\beta_{\|O}$, $\gamma_O = (1 - \beta_{\perp o}^2 - \beta_{\|o}^2)^{-1/2}$ and

$$g(x) = \left[\frac{\cos\left(\frac{\pi x}{2}\right)}{(x^2 - 1)}\right]^2. \quad (7)$$

The radius to be used in evaluating y here is the Larmor radius r_L. Primes represent differentiation with respect to the argument. The derivation of the efficiency and a detailed study of η about the mth harmonic is published elsewhere [12].

In (6), there are two strong interactions presented when $m \gg 1$. The relativistic cyclotron maser interaction gives rise to the term proportional to $J_m'(y)g'(x)$. A second major interaction comes from the tendency of those electrons which gain energy to spiral outwards and for those which lose energy to spiral inwards. This effect is nonrelativistic in nature. When E_θ is an increasing function of radius at the beam position, i.e., $J_m''(y) > 0$, one finds

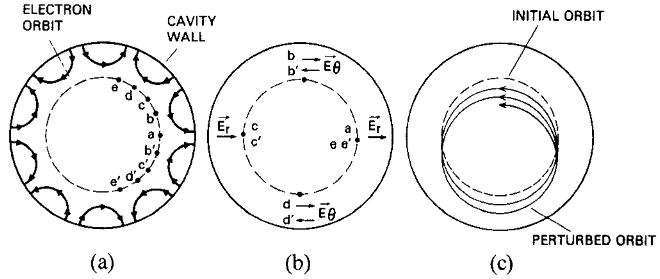

Fig. 1. (a) Electron motion relative to a rotating TE_{511} RF field. An electron initially at a moves, in the frame of the rotating RF field, through points $b-e$ or $b'-e'$ during one cyclotron orbit if it is, respectively, at the $(m-1)$th or $(m+1)$th harmonic. (b) Corresponding lab frame positions and RF electric-field vector. (c) Lab frame weak-field linear perturbed orbit at the $(m-1)$th harmonic.

enhanced energy gain for those electrons which gain energy and decreased loss for those which lose energy. The net effect on the beam is energy gain and a damping of the RF field. This interaction gives rise to the term proportional to $J_m''(y)g(x)$ in (6). Cyclotron maser emission will be unefected by absorption from this term as long as $\beta_{\perp O}^2\omega/k_{\|}\beta_{\|O} \gg m$.

The peniotron-like interaction occurs at the $(m \pm 1)$th harmonics, where $\omega \simeq (m \pm 1)\Omega/\gamma$. At the $(m-1)$th harmonic, electrons move forward through one lobe of the m-fold azimuthal TE_{mnl} field pattern per cyclotron orbit, at the $(m+1)$th harmonic they move backwards through one lobe. Since the electrons spend many cyclotron orbits within the gyrotron cavity, they pass through all phases with respect to the RF field many times. The resonant motion of the electrons through the RF field leads to a net RF electric-field vector. Using $\theta = \theta_O + \Omega t/\gamma_O$, where θ_O gives the electron position angle at $t = 0$, we find the net electric field $\langle\vec{E}\rangle_{m\pm1}$ averaged over a cyclotron orbit at the $(m \pm 1)$th harmonic is:

$$\langle\vec{E}\rangle_{m\pm1} = \frac{E_O \sin k_{\|}z}{2}\left[\frac{mJ_m(y)}{y} \mp J_m'(y)\right] \cdot[\hat{x}\cos(m \pm 1)\theta_O \pm \hat{y}\sin(m \pm 1)\theta_O] \quad (8)$$

where use was made of (1)–(2) with $\omega \gg k_{\|}$. From (8), it is clear that the magnitude of the net electric field is the same for all electrons independent of θ_O, and differs only in its direction. To clarify how this net field arises, we follow a typical electron through its orbit. Fig. 1(a) shows the motion, relative to a rotating TE_{511} RF field, of an electron with $\theta_O = 0$. This electron initially at position a moves through positions $b-e$ during one cyclotron orbit if $\omega \simeq (m-1)\Omega/\gamma$, and moves through positions $b'-e'$ if $\omega \simeq (m+1)\Omega/\gamma$. Fig. 1(b) gives the corresponding lab frame positions, and the RF electric-field vector at these positions. The electric field at the orbital positions chosen is either entirely due to E_r or E_θ. At the $(m-1)$th harmonic, the E_r and E_θ components of the RF field give rise to net fields which are in the same direction. At the $(m+1)$th harmonic, the E_r and E_θ components give rise to net fields in opposite directions. The direction of the vector $\langle\vec{E}\rangle_{m\pm1}$ for an electron with $\theta_O \neq 0$ is determined by the direction

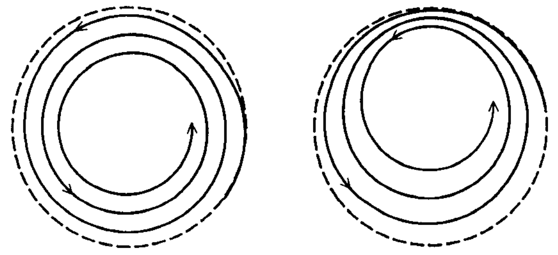

Fig. 2. (a) Typical orbit of an electron rapidly losing energy due to the mth harmonic cyclotron maser interaction. (b) Typical orbit of an electron rapidly losing energy due to the $(m-1)$th harmonic peniotron-like interaction.

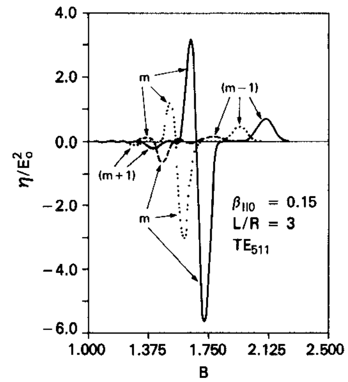

Fig. 3. Efficiency as a function of dimensionless magnetic field. For the solid curve $\gamma_O = 1.3$, for the dotted curve $\gamma_O = 1.2$, and for the dashed curve $\gamma_O = 1.1$. Emission corresponds to positive efficiency.

of $\vec{E}_r$ in the lab frame at the time when this electron is at position a in Fig. 1(a). The effect of the net electric field on the electrons is to cause them to drift at right angles to $\langle \vec{E} \rangle_{m\pm1}$ and to the background magnetic field $B_O \hat{z}$. Fig. 1(c) shows the perturbed orbit for the $\theta_O = 0$, $(m-1)$th harmonic case. There is a similar drift of comparable magnitude due to the axial RF magnetic field. This magnetic drift is always in the direction opposite to drift due to the net electric field.

For the initially axis-centered beam, $\vec{v} \cdot \vec{E}$ averages to zero over each cyclotron orbit, as is clear from Fig. 1(b), and there is no net emission or absorption of RF radiation. When the beam electrons have drifted off-axis as they move through the gyrotron cavity, this is no longer true. Now the electrons move asymmetrically through the RF fields, and we find emission at the $(m-1)$th harmonic, and absorption at the $(m+1)$th harmonic. Fig. 2 shows examples of nonlinear orbits for the cyclotron maser mth harmonic and peniotron-like $(m-1)$th harmonic. The peniotron-like interaction perturbed orbits are such that each electron moves in a near-identical manner with respect to the RF field. Thus, when emission occurs, all electrons lose nearly the same energy, unlike the case for cyclotron maser interaction. For the cyclotron maser interaction, azimuthal bunching leads to only a net energy loss, most electrons losing energy but some gaining energy.

We present here the efficiency in the linear regime for fast-wave interactions near the $(m \pm 1)$th harmonics. We find at the $(m \pm 1)$th harmonic

$$\eta \simeq \frac{E_O^2}{2\gamma_O (k_\parallel \beta_{\parallel O})^2} \frac{[J_{m\pm1}(y) - \beta_{\perp O} J_m(y)]}{(\gamma_O - 1)} \cdot \left[m J_m'(y) - \frac{m J_m(y)}{y} \mp (m \pm 1) \beta_{\perp O} J_m''(y) \right] g(x^\pm) \quad (9)$$

where $x^\pm = [(m \pm 1)\Omega/\gamma_O - \omega]/k_\parallel \beta_{\parallel O}$ and, as before, $y = X_{mn} r_L$. The term $J_{m\pm1} = m J_m / y \mp J_m'$ in the first factor comes from the drift due to the RF electric field. The second $\beta_{\perp O} J_m$ term in the first factor comes from the drift due to the RF magnetic field. At the $(m-1)$th harmonic, the first factor is positive, at the $(m+1)$th harmonic it is negative. The second factor in (9) is positive at both $(m \pm 1)$th harmonics, but larger at the $(m-1)$th. Equation (9) comes from taking the limit of $|(m \pm 1)\Omega/\gamma_O - \omega| \ll \omega$ in the general formula for η given in [12].

In Fig. 3, we show the efficiency in the linear regime as a function of B_O covering the $(m-1)$th, mth, and $(m+1)$th harmonics, for $\beta_{\parallel O} = 0.15$, $L/R = 3$, and several values of γ_O. Emission is strongest for the cyclotron maser interaction, except at the lowest value of γ_O, where the $(m-1)$th harmonic peniotron-like interaction is larger. In Figs. 4–5 we show, for $\gamma_O = 1.3$ and $\gamma_O = 1.1$, respectively, the start oscillation beam power QP_b as a function of magnetic field for TE_{m11} and TE_{m21} modes. Here Q is the quality factor. The higher $(n > 2)$ TE_{mn1} modes were found to have start oscillation beam powers too high to be important in the given magnetic-field range. The several branches for TE_{111}, TE_{211}, and other modes occur at the maxima of $g'(x)$. From Fig. 4, we see that, for moderate values of γ_O, several of the TE_{m11} $(m-1)$th harmonic peniotron-like modes are accessible with no mode competition, namely TE_{511}, TE_{411}, and TE_{311}. If all odd modes can be supressed, say by making a sever in the cavity, then the TE_{811} $(m-1)$th harmonic mode is also accessible. For the smaller $\gamma_O = 1.1$ value in Fig. 5, the rapid rise in start oscillation beam power with increasing m, and the relative rise in beam power at the same mode of the cyclotron maser interaction over the peniotron-like interaction leads to less mode competition from TE_{m21} mth harmonic modes. The rapid rise in beam power with m also implies that the start oscillation beam powers for the TE_{511}, $TE_{611} \cdots (m-1)$th harmonic modes would probably be unreasonably high.

In the linear limit we have shown that, except for small γ_O, the efficiency for the peniotron-like $(m-1)$th harmonic interaction is less than that for the cyclotron maser interaction. In the high RF field nonlinear limit, this is no longer true. We have calculated efficiencies in the high-field limit, optimizing the energy loss as a function of RF field amplitude and background magnetic field. The beam electron dynamics were solved for numerically using the single-mode approximation and ignoring space charge. Reference [12] gives a detailed discussion of the equations and method used. The resulting efficiencies for a TE_{511} mode gyrotron with $\beta_{\parallel O} = 0.15$ and $L/R = 3$ are shown in Fig. 6 as a function of γ_O. Efficiencies for the peniotron-like interac-

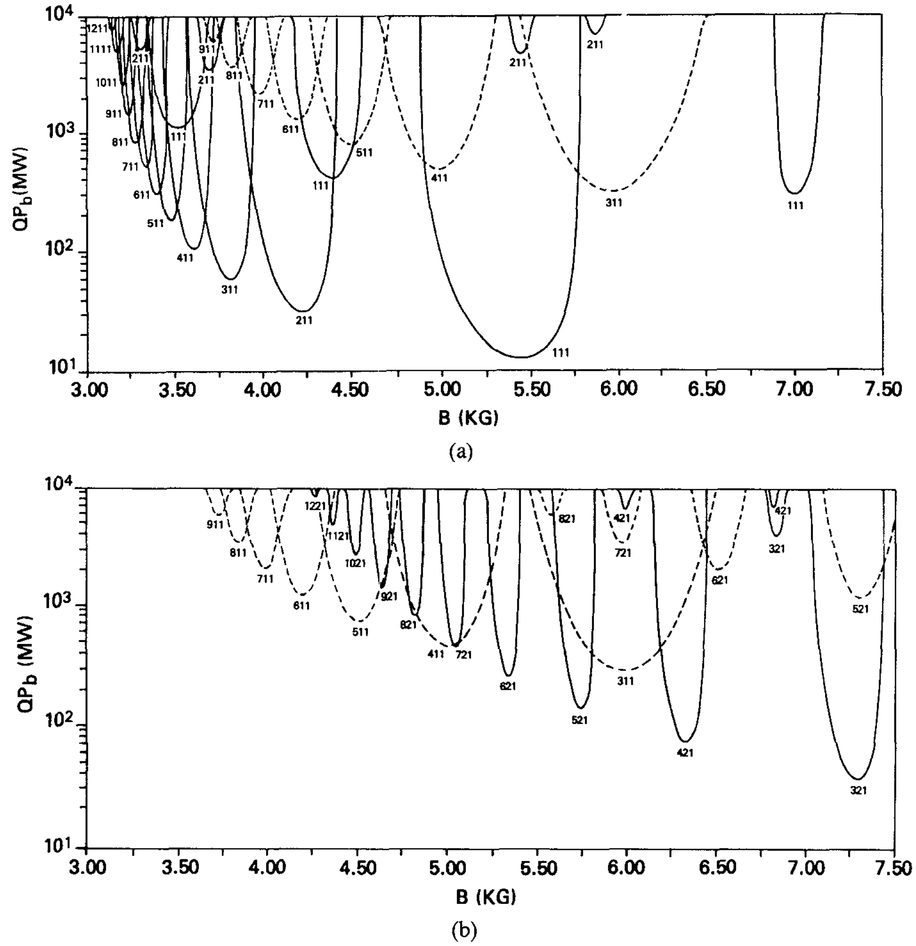

Fig. 4. (a) Start oscillation beam power for TE_{m11} modes as a function of magnetic field (in Kilogauss) for $\beta_{\|O} = 0.15$, $L/R = 3$, $\gamma_O = 1.3$. The dashed curves are from the $(m-1)$th harmonic peniotron-like interaction, while the solid curves are from the mth harmonic cyclotron resonant interaction. (b) Start oscillation beam power for TE_{m21} modes plus those from the $(m-1)$th harmonic TE_{m11} peniotron-like interaction.

tion were found to be $\lesssim 65$ percent, while, for the cyclotron maser interaction, efficiencies were only $\lesssim 25$ percent. For small γ_O, where mode competition may be less of a problem for the peniotron-like modes, the difference in efficiencies is largest.

As mentioned above, all electrons in the beam undergo near identical histories for the peniotron-like interaction. Typically, the standard deviation of the change in electron kinetic energy is a few percent of the average change in kinetic energy for the $(m-1)$th harmonic case. The standard deviation of the change in electron kinetic energy may however be as large as the average change in kinetic energy itself for the cyclotron maser interaction. It should therefore prove easier to further enhance the already high peniotron-like efficiency by tapering the cavity or magnetic field than it would be for the cyclotron maser. Use of a depressed collector on the cold beam produced by the peniotron-like interaction may also lead to higher net efficiencies.

In conclusion, we have given the efficiency in the weak field fast-wave limit for the mth harmonic cyclotron maser and $(m \pm 1)$th harmonic peniotron-like interaction for a TE_{mnl} whispering gallery gyrotron oscillator. Start oscillation beam power curves show that the $(m-1)$th harmonic emission feature can be accessible, even though there is some mode competition. At high RF fields, the peniotron-like interaction proves much more efficient by a factor of 2–3 than the cyclotron maser interaction, with efficiencies $\lesssim 65$ percent for the cases studied. Devices using the peniotron-like interaction may therefore prove capable of high microwave power output.

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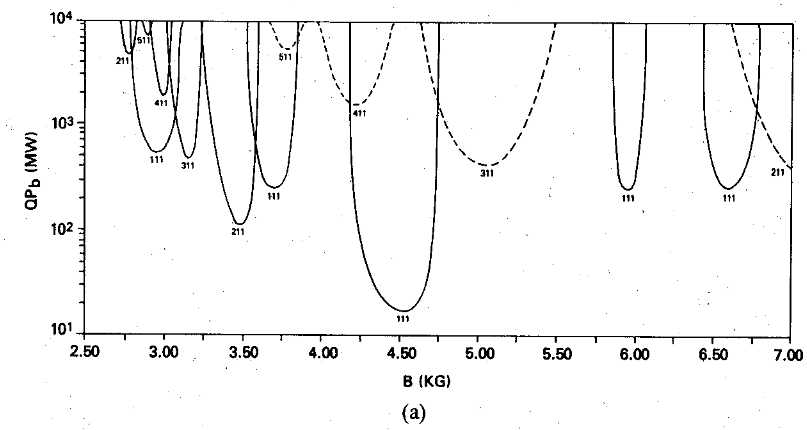

Fig. 5. (a) Start oscillation beam power for TE_{m11} modes as a function of magnetic field (in Kilogauss) for $\beta_{\parallel O} = 0.15$, $L/R = 3$, $\gamma_O = 1.1$. The dashed curves are from the $(m-1)$th harmonic peniotron-like interaction, while the solid curves are from the mth harmonic cyclotron resonant interaction. (b) Start oscillation beam power for TE_{m21} modes plus those from the $(m-1)$th harmonic TE_{m11} peniotron-like interaction.

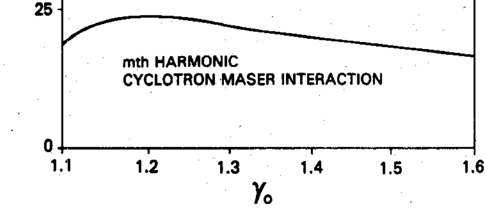

Fig. 6. Optimized efficiency in percent as a function of γ_O for the cyclotron maser and $(m-1)$th harmonic peniotron-like interactions.

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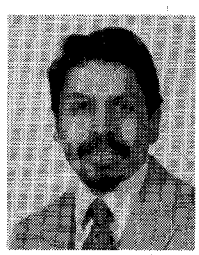

Peter Vitello was born in Glendale, CA, on September 15, 1950. He received the B.S. degree in physics in 1972 from the University of Southern California, Los Angeles, and the Ph.D. degree in theoretical physics in 1977 from Cornell University, Ithaca, NY.

He is a member of the Plasma Physics Division at Science Applications, Inc., McLean, VA. He is currently working on high-harmonic gyrotron oscillators and gyro-klystron amplifiers.

Dr. Vitello is a member of the American Physical Society and of the American Astronomical Society.